

Comparative Analysis of Nuclear Binding, Separation Energies, and
Alpha-Decay Q-Values in Radium and Uranium Isotopic Chains

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Abstract

Understanding how nuclear stability varies along isotopic chains is an important part of nuclear-structure studies. In this work, the semi-empirical mass formula (SEMF) is used to calculate nuclear binding energies for selected radium ($Z = 88$) and uranium ($Z = 92$) isotopes. One-neutron separation energies (S_n), two-neutron separation energies (S_{2n}), and alpha-decay Q-values (Q_α) are derived from the calculated binding energies and compared with evaluated experimental data from the AME2020 atomic mass evaluation. The analysis covers $^{210-233}\text{Ra}$ and $^{218-240}\text{U}$, corresponding to neutron numbers $N = 122-145$ for Ra and $N = 126-148$ for U. The SEMF reproduces the smooth binding-energy systematics reasonably well, but it does not reproduce the localized shell structure visible in separation energies. For Ra, the mean absolute errors are about 0.38 MeV for S_n , 0.76 MeV for S_{2n} , and 1.60 MeV for Q_α ; for U, the corresponding values are about 0.20, 0.29, and 0.85 MeV when AME2020 estimated entries are excluded from the error statistics. The $N = 126$ region provides a clear illustration of the limitation: the experimental two-neutron shell-gap indicator is substantially larger than the smooth SEMF prediction. The study also shows that the alpha-decay energy is sensitive to the balance between binding changes in the parent and daughter nuclei and to the increasing Coulomb contribution with atomic number. The originally proposed interpretation of an inverse separation-energy/ Q_α relationship is

therefore treated carefully: the chain-wise Pearson correlations are positive because both quantities have a strong dependence on mass number, whereas correlations of successive changes show a strong local anti-correlation for S_{2n} and $Q\alpha$ (approximately $r = -0.93$ for Ra and -0.96 for U). The results confirm that SEMF is useful for broad trends in heavy nuclei but requires shell and deformation corrections for quantitative description of localized structure.

Keywords: Semi-empirical mass formula, nuclear binding energy, separation energy, alpha-decay Q-value, radium isotopes, uranium isotopes.

التحليل المقارن لطاقة الربط النووية، وطاقات الفصل، وقيم طاقة تفكك ألفا Q في السلاسل النظرية للراديوم واليورانيوم

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الملخص

فهم كيفية تغيير الاستقرار النووي على طول السلاسل النظرية يُعد جزءاً مهماً من دراسات البنية النووية. في هذا العمل، تُستخدم الصيغة شبه التجريبية للكتلة (SEMF) لحساب طاقات الترابط النووية لنظائر مختارة من الراديوم ($Z = 88$) واليورانيوم ($Z = 92$). تُشتق طاقات فصل نيوترون واحد (S_n)، وطاقات فصل نيوترونين (S_{2n})، وقيم Q لتحلل ألفا ($Q\alpha$) من طاقات الترابط المحسوبة، وتُقارن بالبيانات التجريبية المقيمة من تقييم الكتلة الذرية AME2020. يغطي التحليل النظائر $^{210}_{88}\text{Ra}$ و $^{218}_{88}\text{Ra}$ ، وهو ما يقابل أعداد نيوترونات $N = 122-145$ للراديوم و $N = 126-148$ لليورانيوم. تُعيد الصيغة SEMF إنتاج الانتظام السلس لطاقة الترابط بشكل جيد نسبياً، لكنها لا تُعيد إنتاج البنية القشرية الموضعية الظاهرة في طاقات الفصل. بالنسبة للراديوم يبلغ متوسط الخطأ المطلق نحو 0.38 MeV لـ S_n ، و 0.76 MeV لـ S_{2n} ، و 1.60 MeV لـ $Q\alpha$ ؛ أما بالنسبة لليورانيوم، فالقيم المقابلة هي نحو 0.20 و 0.29 و 0.85 MeV ، وذلك بعد استبعاد المدخلات المقدرة في AME2020 من إحصاءات الخطأ. تُقدّم منطقة

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$N = 126$ توضيحاً واضحاً لهذا القصور: إذ إن مؤشر الفجوة القشرية التجريبي لفصل نيوترونين أكبر بكثير من التنبؤ السلس لصيغة SEMF كما تُظهر الدراسة أن طاقة تحلل ألفا حساسة للتوازن بين تغيرات الترابط في النواة الأم والنواة الوليدة، ولإسهام كولوم المتزايد مع العدد الذري. لذلك، تُعامل الفكرة التفسيرية التي طُرحت في الأصل حول وجود علاقة عكسية بين طاقة الفصل و Q_α بحذر: فارتباطات بيرسون على مستوى السلسلة موجبة لأن كلا الكميتين تعتمدان بقوة على العدد الكتلي، في حين أن ارتباطات التغيرات المتتالية تُظهر ارتباطاً عكسياً موضعياً قوياً بين S_{2n} و Q_α بقيمة تقريبية $r = -0.93$ للراديوم و $r = -0.96$ لليورانيوم. تؤكد النتائج أن صيغة SEMF مفيدة لوصف الاتجاهات العامة في النوى الثقيلة، لكنها تتطلب تصحيحات قشرية وتشويهية للوصف الكمي للبنية الموضعية.

الكلمات المفتاحية: معادلة الكتلة شبه التجريبية، طاقة الربط النووية، طاقة الفصل، قيمة طاقة تفكك ألفا، نظائر الراديوم، نظائر اليورانيوم.

1. Introduction

The nuclear binding energy is one of the most useful macroscopic indicators of nuclear stability. It measures the energy required to separate a nucleus into its constituent protons and neutrons and therefore provides a direct connection between nuclear masses, decay energetic, and the energetically preferred configurations of nuclei. For an isotopic chain, however, the absolute binding energy changes smoothly while its finite differences can reveal much more localized information about shell structure and pairing.

The semi-empirical mass formula (SEMF), also known as the Bethe–Weizsäcker mass formula, provides a compact liquid-drop description of nuclear binding. Its five principal contributions represent the volume, surface, Coulomb, asymmetry, and pairing terms (Huang et al., 2021; Wang et al., 2021). The model is deliberately macroscopic: it captures the average behavior of nuclei but does not explicitly contain shell corrections, deformation effects, or other microscopic structure. This makes it especially useful for studying broad trends while also making its limitations clear in regions close to shell closures.

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Separation energies are finite differences of nuclear binding energies. The one-neutron separation energy (S_n , where n represent a single neutron) measures the energy needed to remove one neutron, whereas the two-neutron separation energy (S_{2n} , where $2n$ represent a pair neutrons) measures the energy needed to remove two neutrons. The latter usually gives a smoother indicator of shell structure because the leading odd-even pairing oscillation is reduced. Alpha-decay Q-values are similarly related to differences in binding between a parent nucleus, its daughter, and an alpha particle. For heavy nuclei, the Coulomb energy is particularly important because it grows strongly with proton number and contributes to the tendency toward alpha decay and, ultimately, fission (Buskirk et al, 2024).

The present study applies a fixed, standard SEMF coefficient set to selected Ra and U isotopic chains and compares the calculated binding and separation quantities with AME2020. AME2020 provides atomic masses and derived reaction energies, including S_{2n} , Q_α , and S_n , in its recommended data files (Huang et al., 2021; Wang et al., 2021). The analysis is designed to answer three questions: (i) how accurately does a standard SEMF reproduce smooth binding and separation-energy trends in this heavy mass region; (ii) where do shell and pairing effects produce systematic departures from the SEMF; and (iii) how does the relationship between neutron separation and alpha-decay energetics differ between $Z = 88$ and $Z = 92$ (Wu & Zhao ,2024).

2. Theoretical Background

2.1 Semi-Empirical Mass Formula

The SEMF treats the nucleus as a finite drop of nuclear matter. The binding energy is written as the sum of five macroscopic contributions:

$$B(A, Z) = a_v A - a_s A^{\frac{2}{3}} - a_c Z(Z-1)A^{-\frac{1}{3}} - a_a \frac{(A-2Z)^2}{A} + \delta.$$

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The $Z(Z-1)$ form is used for the Coulomb term because it represents the number of proton-proton pairs and is commonly used in SEMF calculations (Wang et. El, 2021).

The physical origin of each term is summarized below.

Volume term, $a_v A$: Owing to the short range of the nuclear force, each nucleon interacts effectively with only its nearest neighbors rather than with the nucleus as a whole. Consequently, the binding energy scales approximately linearly with A , analogous to the extensive property of bulk matter.

Surface term, $a_s A^{\frac{2}{3}}$: Nucleons located at the nuclear surface possess fewer neighboring nucleons than those in the interior and are therefore less strongly bound. This term, proportional to the surface area of the nuclear "drop," reduces the binding energy predicted by the volume term alone, in direct analogy to surface tension in a classical liquid.

Coulomb term, $a_c Z(Z-1)A^{-\frac{1}{3}}$: This term accounts for the electrostatic repulsion among protons, which acts in opposition to the attractive nuclear force. Its dependence on $(Z-1)$ renders it increasingly significant with increasing atomic number, and it is the principal term responsible for the progressive neutron excess ($N > Z$) observed in heavy nuclei, including the radium and uranium isotopes considered in this study.

Asymmetry term, $a_a \frac{(A-2Z)^2}{A}$: Arising from the Pauli exclusion principle, this term penalizes deviations from $N=Z$, since an imbalance between proton and neutron numbers forces nucleons to occupy higher, less energetically favorable quantum states. In heavy nuclei, this term operates in competition with the Coulomb term, the balance of which determines the location of the line of β -stability.

Pairing term, δ : This term is a semi-empirical correction accounting for the additional binding energy associated with the coupling of like nucleons into spin-zero pairs (Zhang, H.-F.2024). The pairing contribution is

$$\delta = +a_p/A^{1/2} \text{ (even-even); } 0 \text{ (odd-A); } -a_p/A^{1/2} \text{ (odd-odd).}$$

Where a_p represent the pairing energy constant and $A^{1/2}$ accounts for the mass dependence of nucleon pairing interactions.

The coefficients a_i (where $i \in (v, s, c, a, p)$) denotes the respective volume, surface, coulomb, asymmetry, and paring coefficient terms) used in this study:

Volume term (a_v)= 15.75 MeV

Surface term (a_s)=17.08 MeV

Coulomb term (a_c) = 0.711 MeV

Asymmetry term (a_a) = 23.70 MeV

Pairing term (a_p) = 11.18 MeV

2.2 Neutron Separation Energies

The one-neutron separation energy is

$$S_n(A, Z) = B(A, Z) - B(A-1, Z).$$

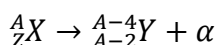
The two-neutron separation energy is

$$S_{2n}(A, Z) = B(A, Z) - B(A-2, Z).$$

S_{2n} is especially useful for identifying shell structure because the alternating odd-even component is reduced relative to S_n . A two-neutron shell-gap indicator can be defined as $\delta_{2n}(N) = S_{2n}(N) - S_{2n}(N+2)$. A large positive value is a signature of an abrupt change in the neutron shell structure.

2.3 Alpha-Decay Q-Value

The general alpha decay is



Where the alpha particle is $\alpha = {}^4_2He$

The alpha- decay energy is

$$Q_\alpha = B(A-4, Z-2) + B_\alpha - B(A, Z).$$

Because the SEMF is not designed to describe the strongly clustered, very light 4He nucleus accurately, the experimental alpha-particle binding energy $B_\alpha = 28.295674$ MeV is used in the calculated Q_α expression. This avoids introducing a known light-nucleus limitation into the heavy-nucleus Q_α comparison.

3. Methodology

3.1 Isotopic Chains and Data Selection

The analysis uses $^{210-233}\text{Ra}$ ($Z = 88$, $N = 122-145$) and $^{218-240}\text{U}$ ($Z = 92$, $N = 126-148$). The lower uranium limit is selected because AME2020 provides the relevant separation-energy quantities from ^{218}U onward without requiring the use of non-calculable entries for this study. Entries explicitly marked as estimated in AME2020 are retained in the numerical tables and plots but are excluded from the error statistics for the affected quantity. This is important because AME2020 distinguishes evaluated experimental information from values estimated from trends or other constraints [3], [4].

3.2 Numerical Procedure

- Set Z and A for each isotope and calculate $N = A - Z$.
- Evaluate the five SEMF terms and the pairing contribution.
- Calculate total binding energy $B(A, Z)$ and binding energy per nucleon B/A .
- Calculate S_n from $B(A, Z) - B(A-1, Z)$.
- Calculate S_{2n} from $B(A, Z) - B(A-2, Z)$.
- Calculate Q_α from $B(A-4, Z-2) + B_\alpha - B(A, Z)$.
- Retrieve the corresponding AME2020 values of binding energy, S_n , S_{2n} , and Q_α .
- Compare calculated and AME2020 values using absolute error, mean absolute error (MAE), root-mean-square error (RMSE), mean signed error, and mean absolute percentage error (MAPE).

4. Results and Discussion

4.1 Binding Energy

The SEMF reproduces the overall magnitude and smooth evolution of the total binding energy in both isotopic chains. However, the calculated binding energies are systematically higher than the AME2020 values over the selected ranges. This systematic deviation is expected because the standard SEMF provides a smooth macroscopic description of nuclear binding and does not explicitly include microscopic shell, deformation, and other local structural corrections. In the heavy-mass region considered here, these effects

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can produce localized deviations from the smooth liquid-drop trend, particularly near shell closures and regions of enhanced nuclear structure. In addition, the adopted SEMF coefficients represent a global parameterization rather than a fit specifically optimized for the Ra and U isotopic chains. Consequently, a small systematic bias in the calculated total binding energy is expected.

The MAE is 10.32 MeV for Ra and 12.38 MeV for U, corresponding to approximately 0.60% and 0.71%, respectively, of the total binding energy. Although these relative errors are small because the total binding energies of heavy nuclei are large, the absolute residuals of several MeV should not be considered negligible. This distinction is important because separation energies and decay Q-values are calculated from differences between neighboring binding energies and are therefore more sensitive to local deviations in the mass surface.

Here, A denotes the mass number, n the neutron number, B/A the binding energy per nucleon, and $B_{\text{SEMF}} - B_{\text{AME2020}} = \Delta B$ the difference between the calculated and evaluated binding energies. All energies are reported in MeV table (1).

Table 1. Comparison of calculated and AME2020 binding energy per nucleon for Ra isotopes

A	N	B_{SEMF} (MeV)	B_{AME2020} (Mev)	ΔB	A	N	B_{SEMF}	B AME2020	ΔB
210	122	7.777	7.741	+0.035	222	134	7.747	7.697	+0.050
211	123	7.775	7.741	+0.033	223	135	7.737	7.685	+0.051
212	124	7.779	7.748	+0.031	224	136	7.733	7.680	+0.053
213	125	7.775	7.746	+0.029	225	137	7.722	7.668	+0.054
214	126	7.778	7.749	+0.029	226	138	7.717	7.662	+0.055
215	127	7.773	7.739	+0.033	227	139	7.705	7.648	+0.056
216	128	7.774	7.737	+0.037	228	140	7.699	7.642	+0.056
217	129	7.767	7.727	+0.041	229	141	7.686	7.628	+0.057
218	130	7.767	7.725	+0.042	230	142	7.679	7.622	+0.057
219	131	7.760	7.714	+0.046	231	143	7.665	7.608	+0.057

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A	N	B _{SEMF} (MeV)	B _{AME2020} (Mev)	ΔB	A	N	B _{SEMF}	B _{AME2020}	ΔB
220	132	7.758	7.712	+0.047	232	144	7.657	7.600	+0.057
221	133	7.749	7.701	+0.048	233	145	7.642	7.586	+0.057

Figure 1 shows the variation of the binding energy per nucleon, B/A , with neutron number N for the selected Ra isotopes. The AME2020 and SEMF curves exhibit a generally decreasing trend as the neutron number increases.

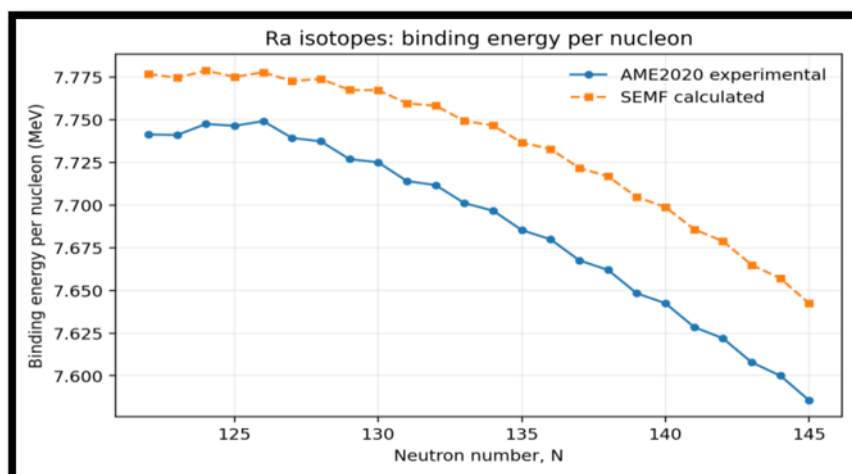


Fig. 1. Comparison of SEMF-calculated and AME2020 binding energy per nucleon, B/A , as a function of neutron number, N , for the selected Ra isotopes.

However, the SEMF curve remains systematically above the AME2020 curve throughout the investigated range, indicating that the SEMF overestimates the binding energy per nucleon for these isotopes. See (Fig.2 & table (2)).

Table 2. U binding energy per nucleon.

A	N	S_n [Calculated]	S_n [Experimental] AME2020	A	N	S_n [Calculated]	S_n [Experimental] AME2020
218	126	7.683	7.641	229	137	7.677	7.621
219	127	7.682	7.636	230	138	7.676	7.621
220	128	7.688	7.640	231	139	7.668	7.613

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A	N	S_n [Calculated]	S_n [Experimental] AME2020	A	N	S_n [Calculated]	S_n [Experimental] AME2020
221	129	7.687	7.635	232	140	7.666	7.612
222	130	7.691	7.638	233	141	7.657	7.604
223	131	7.688	7.632	234	142	7.654	7.601
224	132	7.691	7.635	235	143	7.645	7.591
225	133	7.687	7.630	236	144	7.641	7.586
226	134	7.689	7.632	237	145	7.630	7.576
227	135	7.683	7.626	238	146	7.625	7.570
228	136	7.683	7.627	239	147	7.613	7.559

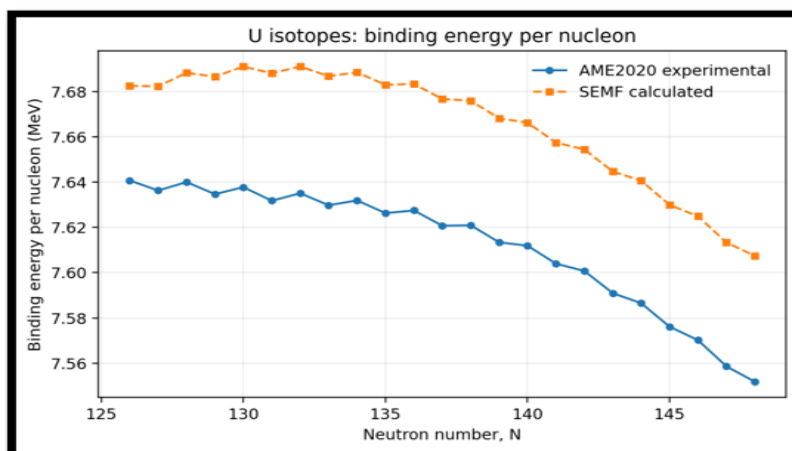


Fig. 2. Energy per nucleon for the selected U isotopes.

4.2 One-Neutron Separation Energy

S_n displays the expected odd-even staggering caused by pairing. The SEMF reproduces the broad downward trend with increasing neutron number but cannot reproduce every local fluctuation. The MAE is 0.38 MeV for Ra and 0.20 MeV for U. The smaller uranium error over the selected range indicates that the fixed coefficient set happens to follow the smooth U separation-energy systematics somewhat more closely than the Ra chain. see table (3) & Fig .3.

For one-neutron separation energy, use:

$$S_n(A, Z) = B(A, Z) - B(A - 1, Z)$$

where:

S_n = one-neutron separation energy

$B(A, Z)$ = binding energy of the parent nucleus

$B(A - 1, Z)$ = binding energy after removing one neutron.

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For the SEMF calculation:

$$S_n^{SEMF}(A, Z) = B_{SEMF}(A, Z) - B_{SEMF}(A - 1, Z)$$

For the AME2020 values

$$S_n^{AME2020} = B_{AME2020}(A, Z) - B_{AME2020}(A - 1, Z)$$

Table 3. Ra one-neutron separation energies.

A	N	S_n^{SEMF}	$S_n^{AME2020}$	A	N	$S_n^{AME2020}$	$S_n^{AME2020}$
210	122	9.008	9.487	222	134	7.156	6.715
211	123	7.301	7.682	223	135	5.516	5.158
212	124	8.678	9.102	224	136	6.876	6.479
213	125	6.982	7.527	225	137	5.246	4.904
214	126	8.356	8.324	226	138	6.603	6.397
215	127	6.672	5.632	227	139	4.984	4.561
216	128	8.044	7.312	228	140	6.338	6.309
217	129	6.371	5.473	229	141	4.728	4.450
218	130	7.740	7.315	230	142	6.079	6.117
219	131	6.078	5.323	231	143	4.479	4.371
220	132	7.444	7.193	232	144	5.826	5.791
221	133	5.793	5.380	233	145	4.236	4.234

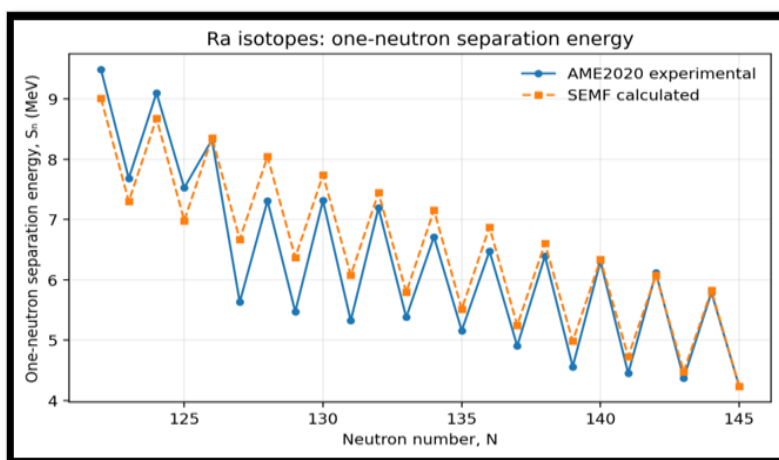


Fig. 3. SEMF and AME2020 one-neutron separation energies for Ra.

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The Comparison of SEMF-calculated and AME2020 one-neutron separation energies, S_n , as a function of neutron number, N , for the selected U isotopes is illustrated on table 4 & Fig.4.

Table 4. Comparison of SEMF-calculated and AME2020 one-neutron separation energies for the selected U isotopes.

A	N	S_n (SEMF) (MeV)	S_n (AME2020) (MeV)	ΔS_n (MeV)	A	N	S_n (SEMF) (MeV)	S_n (AME2020) (MeV)	ΔS_n (MeV)
218	126	9.313	9.147	+0.166	229	137	6.081	6.081	+0.000
219	127	7.637	6.670	+0.967	230	138	7.667	7.667	+0.000
220	128	8.989	8.354	+0.635	231	139	5.880	5.880	+0.000
221	129	7.325	6.565	+0.760	232	140	7.268	7.268	+0.000
222	130	8.674	8.318	+0.356	233	141	5.762	5.762	+0.000
223	131	7.021	6.299	+0.722	234	142	6.845	6.845	+0.000
224	132	8.368	8.374	-0.006	235	143	5.297	5.297	+0.000
225	133	6.725	6.442	+0.283	236	144	6.546	6.546	+0.000
226	134	8.069	8.115	-0.046	237	145	5.126	5.126	+0.000
227	135	6.437	6.355	+0.082	238	146	6.154	6.154	+0.000
228	136	7.778	7.896	-0.118	239	147	4.806	4.806	+0.000

Note. A denotes the mass number, N denotes the neutron number, and S_n is the one-neutron separation energy. The difference is defined as $\Delta S_n = S_n(\text{SEMF}) - S_n(\text{AME2020})$. All energies are reported in MeV.

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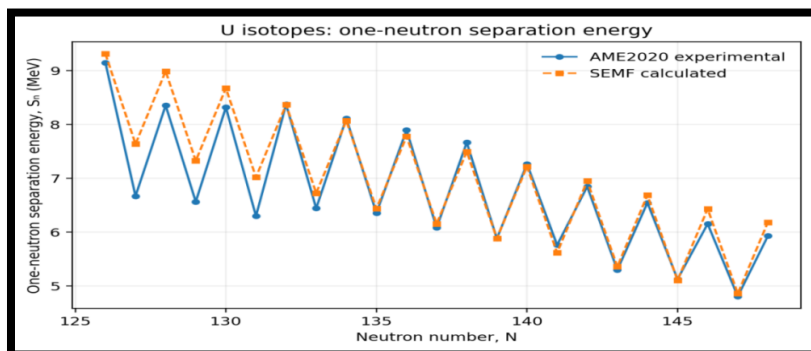


Fig. 4. Comparison of SEMF-calculated and AME2020 one-neutron separation energies, S_n , as a function of neutron number, N , for the selected U isotopes.

4.3 Two-Neutron Separation Energy

S_{2n} provides a clearer view of shell structure because the strongest odd-even oscillation is averaged out. For Ra, the MAE and RMSE are 0.76 and 0.87 MeV, respectively. For U, they are 0.29 and 0.46 MeV. The U chain therefore shows the closest agreement with the smooth SEMF description for S_{2n} in the selected range table & Figs (5,6).

Table 5. Comparison of SEMF-calculated and AME2020 two-neutron separation energies for the selected Ra isotopes.

A	N	SEMF	AME 2020	ΔB	A	N	SEMF	AME 2020	ΔB
210	122	16.637	17.428	-0.791	222	134	12.950	12.095	+0.855
211	123	16.309	17.169	-0.860	223	135	12.673	11.873	+0.799
212	124	15.979	16.784	-0.806	224	136	12.392	11.637	+0.755
213	125	15.660	16.629	-0.969	225	137	12.123	11.383	+0.740
214	126	15.339	15.851	-0.513	226	138	11.850	11.301	+0.549
215	127	15.029	13.956	+1.073	227	139	11.587	10.958	+0.629
216	128	14.716	12.944	+1.772	228	140	11.321	10.870	+0.451
217	129	14.415	12.785	+1.630	229	141	11.066	10.758	+0.307
218	130	14.111	12.789	+1.323	230	142	10.807	10.567	+0.240
219	131	13.818	12.638	+1.180	231	143	10.558	10.488	+0.069

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A	N	SEMF	AME 2020	ΔB	A	N	SEMF	AME 2020	ΔB
220	132	13.523	12.516	+1.006	232	144	10.305	10.162	+0.143
221	133	13.238	12.573	+0.665	233	145	10.063	10.025	+0.038

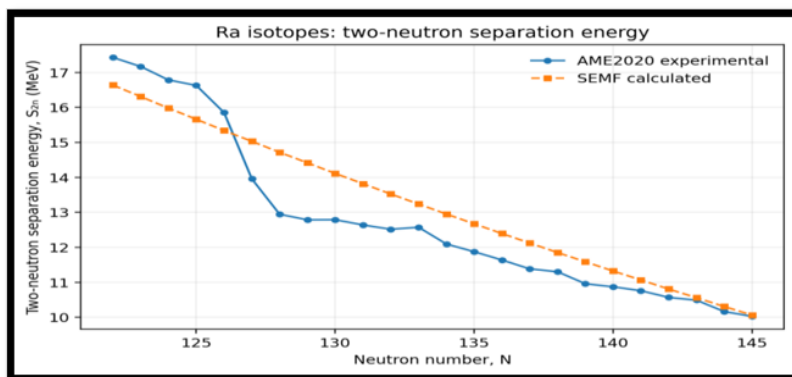


Fig. 5. SEMF and AME2020 two-neutron separation energies for Ra.

Table 6. U two-neutron separation energies.

A	N	SEMF	AME2 020	ΔB	A	N	SEMF	AME2 020	ΔB
218	126	17.271	17.314	-0.044	229	137	13.935	13.977	-0.042
219	127	16.950	15.817	+1.133	230	138	13.652	13.748	-0.096
220	128	16.626	15.024	+1.602	231	139	13.379	13.547	-0.168
221	129	16.314	14.919	+1.395	232	140	13.103	13.148	-0.045
222	130	15.999	14.883	+1.116	233	141	12.837	13.029	-0.192
223	131	15.695	14.617	+1.078	234	142	12.568	12.607	-0.039
224	132	15.388	14.673	+0.716	235	143	12.309	12.143	+0.166
225	133	15.093	14.816	+0.277	236	144	12.047	11.843	+0.204
226	134	14.794	14.557	+0.238	237	145	11.794	11.671	+0.123
227	135	14.506	14.470	+0.037	238	146	11.539	11.279	+0.259
228	136	14.216	14.251	-0.036	239	147	11.292	10.960	+0.332

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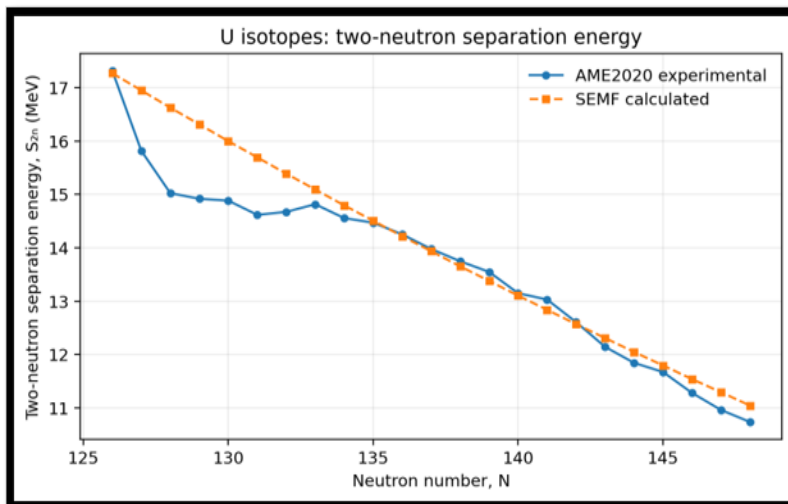


Fig. 6. SEMF and AME2020 two-neutron separation energies for U.

4.3 Alpha-Decay Q-Values

The calculated Q_α values show a systematic negative bias relative to AME2020, with mean signed errors of -1.60 MeV for Ra and -0.73 MeV for U. The MAE is 1.60 MeV for Ra and 0.85 MeV for U. The corresponding MAPE values are 23.3% and 10.4% , respectively. The relatively larger percentage error for Ra partly reflects the lower Q_α values at the neutron-rich end of the selected Ra chain. See Fig7 & table (7).

Table 7. Ra alpha-decay Q-values (MeV)

A	N	SEMF	AME2020	Q_Δ	A	N	SEMF	AME2020	Q_Δ
210	122	6.546	7.151	-0.605	222	134	4.431	6.678	-2.247
211	123	6.370	7.042	-0.672	223	135	4.239	5.979	-1.740
212	124	6.208	7.032	-0.824	224	136	4.061	5.789	-1.728
213	125	6.029	6.862	-0.833	225	137	3.868	5.097	-1.229
214	126	5.863	7.273	-1.409	226	138	3.687	4.871	-1.183
215	127	5.682	8.862	-3.181	227	139	3.493	4.363	-0.870
216	128	5.513	9.526	-4.013	228	140	3.310	4.070	-0.760

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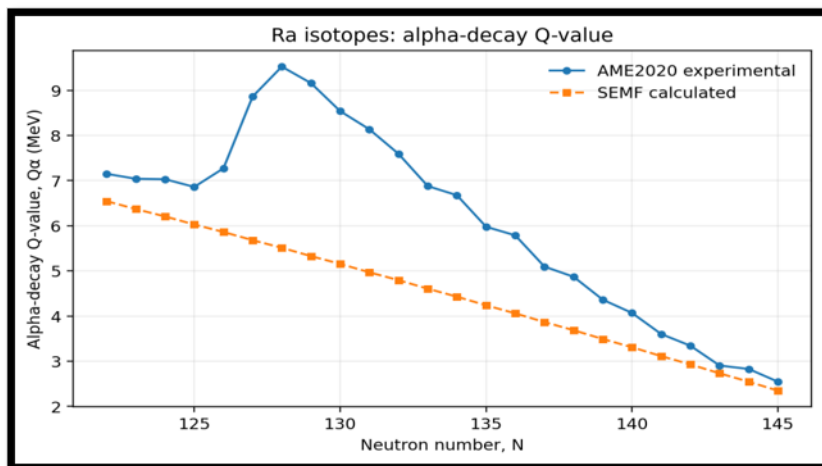


Fig. 7. SEMF and AME2020 alpha-decay Q-values for Ra.

The sudden jump in the experimental data around $A = 215-217$ ($N = 127-129$) stems from nuclear shell structure effects that the standard Semi-Empirical Mass Formula (SEMF) fails to predict. See Fig.8 & table (8).

Table 8. U alpha-decay Q-values.

A	N	SEMF	AME 2020	Δ	A	N	SEMF	AME 2020	Δ
218	126	7.771	8.775	-1.004	229	137	5.937	6.476	-0.539
219	127	7.605	9.950	-2.345	230	138	5.769	5.992	-0.223
220	128	7.452	10.290	-2.838	231	139	5.588	5.576	+0.012
221	129	7.283	9.889	-2.607	232	140	5.419	5.414	+0.005
222	130	7.126	9.481	-2.355	233	141	5.235	4.909	+0.327
223	131	6.954	9.158	-2.203	234	142	5.064	4.858	+0.207
224	132	6.795	8.628	-1.834	235	143	4.879	4.678	+0.201
225	133	6.620	8.007	-1.387	236	144	4.706	4.573	+0.133
226	134	6.458	7.701	-1.243	237	145	4.519	4.234	+0.286
227	135	6.281	7.235	-0.954	238	146	4.345	4.270	+0.075
228	136	6.116	6.800	-0.684	239	147	4.157	4.130	+0.027

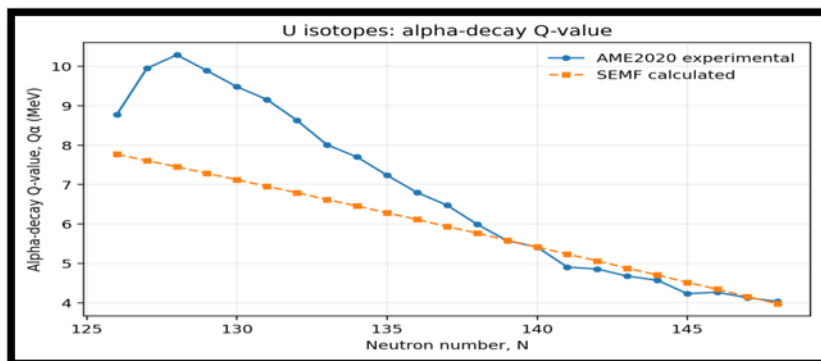


Fig. 8. SEMF and AME2020 alpha-decay Q-values for U.

4.4. Correlation Between Separation Energies and $Q\alpha$

A simple correlation coefficient calculated directly from all isotopes gives positive correlations between S_{2n} and $Q\alpha$ ($r \approx 0.61$ for Ra and $r \approx 0.89$ for U). This should not be interpreted as evidence for a direct causal positive relationship, because both observables contain a strong systematic dependence on mass number. To examine the local relationship between changes along an isotopic chain, the Pearson coefficient was also calculated between successive differences ΔS and $\Delta Q\alpha$. For S_{2n} , this incremental correlation is strongly negative, approximately -0.93 for Ra and -0.96 for U. This supports the qualitative idea that abrupt local reductions in neutron-binding systematics can be associated with increases in alpha-decay energetics, while also showing that the relationship is not a simple global one-to-one inverse law. See table.9.

Table 9: Pearson correlation coefficients (r) between separation energies (S_n , S_{2n} and alpha-decay Q-values for radium (Ra) and uranium (U) isotopic chains, comparing direct level correlations with incremental successive changes.

Chain	Quantity	r (levels)	r (successive changes)
Ra	S_n	0.497	0.023
Ra	S_{2n}	0.609	-0.934
U	S_n	0.622	-0.062
U	S_{2n}	0.887	-0.964

The main strength of SEMF in the present study is its ability to reproduce the smooth macroscopic scale of nuclear binding. The binding-energy-per-nucleon curves remain close to the AME2020 systematics across both chains. This is consistent with the liquid-drop interpretation: the dominant volume, surface, Coulomb, and asymmetry contributions vary relatively smoothly with A and Z .

The largest qualitative limitation is the absence of microscopic shell corrections. At $N = 126$, the experimental S_{2n} behavior contains an abrupt change that is much stronger than the SEMF prediction. Pairing is included explicitly in SEMF, so the model reproduces much of the odd-even staggering in S_n , but pairing alone cannot create the shell gap.

The $Q\alpha$ comparison is also informative. Because $Q\alpha$ is a difference of three binding energies, small systematic errors in each binding energy can accumulate. In the present calculation, the use of the experimental ${}^4\text{He}$ binding energy avoids a separate failure of SEMF for the alpha particle, but the heavy parent and daughter nuclei are still described only by the liquid-drop expression. The resulting negative bias in $Q\alpha$ shows that macroscopic binding alone is not sufficient for high-precision alpha-decay energetics.

The comparison between Ra and U illustrates the increasing importance of the Coulomb contribution with Z . The uranium chain has a generally stronger Coulomb penalty than the radium chain, and this affects both the absolute binding systematics and the alpha-decay energetics. Nevertheless, the observed differences in statistical accuracy should not be attributed to Coulomb effects alone; isotope coverage and local nuclear structure also contribute. Finally, the study demonstrates the importance of separating global trends from local structure. A model can have a sub-percent error in total binding energy while simultaneously missing a shell-gap feature of several MeV in a finite-difference observable. For nuclear-structure applications, separation energies are therefore often more diagnostic than total binding energies.

5. Conclusions

This study provides a consistent SEMF-based analysis of binding energies, one-neutron separation energies, two-neutron separation energies, and alpha-decay Q-values for selected radium and

uranium isotopes. The main conclusions are: (1) SEMF reproduces the smooth heavy-nucleus binding-energy scale well, with total-binding-energy percentage errors below 1% for both chains; (2) the model reproduces S_n and S_{2n} trends reasonably well, with smaller errors for U than Ra over the selected intervals; (3) the $N = 126$ region clearly exposes the missing shell correction, as the experimental S_{2n} shell gap is several times larger than the SEMF prediction; (4) $Q\alpha$ is systematically underpredicted by the present SEMF implementation, although the agreement is better for U; (5) a direct global correlation between separation energies and $Q\alpha$ is affected by the common dependence on neutron number, whereas successive changes in S_{2n} and $Q\alpha$ show a strong local anti-correlation; and (6) $N = 152$ should not be presented as directly validated by the selected Ra/U AME2020 data, although SEMF-only extrapolations can be used to motivate future investigation. Overall, SEMF is a strong starting point for identifying broad nuclear-stability trends, but shell-corrected models are required to explain localized structure and achieve high-precision decay-energy predictions.

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